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Relativistic Mean Field Theory of Surface Pion Condensation in Finite Nuclei

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We study the possible occurrence of surface pion condensation in finite nuclei in the relativistic mean field (RMF) theory. We are led to this conjecture due to the essential role of pions in few-body systems and the recent (p,n) experiments performed at RCNP for spin-isospin excitations of medium and heavy nuclei. We calculate explicitly various N=Z closed shell nuclei with finite pion mean field in the RMF framework and demonstrate the actual occurrence of surface pion condensation.

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The pion is conjectured by Yukawa as the mediator of nucleon-nucleon interaction [1]. The pion is identified then as the Nambu-Goldstone mode when the underlying symmetry, the chiral symmetry, is spontaneously broken [2]. The pion plays the central role in hadron physics, particularly for the low energy phenomena, and now the chiral perturbation approach with the pion as the essential degree of freedom is the powerful tool for the study of hadron properties and collisions.

On the other hand, since the establishment of the shell model, we usually solve nuclear many-body problems in the model space, where single particle states have good parities. Because the parity of a nucleon changes when it absorbs or emits a pion, we must include the higher configurations like 2p-2h (2 particle–2 hole), . . . , to incorporate the effect of the pion in the parity-conserved space. To avoid such complications we renormalize the central and the spin-orbit interactions in nuclei to incorporate the effect of the strong correlations caused by the pion. It means that we make a model space in which the parity is conserved and then define the effective interaction acting between nucleons in the model space. One of the main purpose of the present study is to expand the model space for the pion to play its important role explicitly and see the effect on nuclear structure.

As for the importance of the pion, we remind ourselves of the findings of few-body systems, where they are solved rigorously without the restriction of model space with a realistic nucleon-nucleon interaction. The calculations are performed in the non-relativistic framework and hence the pion is hidden mainly in the tensor force, which is much larger than the central force in the one pion exchange interaction even up to the Compton length. There are many extensive calculations using the variational principle and sophisticated numerical techniques and the calculated results compare with experiments very well [4, 5]. The calculated results demonstrate a dominant role of the tensor force in the few-body system. Almost a half of the attraction is caused by the tensor force, and hence the pion exchange interaction [6]. The recent variational calculations of the Argonne group up to A=8 system also demonstrate the dominant role of the pion and further the importance of the three-body interaction, which also originates from the pion [7].

There are several experimental data demanding the important role of pion in medium and heavy nuclei. The (p,n) reactions on medium and heavy nuclei demonstrate that only a half of the Gamow-Teller ($\sigma\tau$) strengths is carried by 1p-1h excitations, while the rest is interpreted as carried by 2p-2h excitations due to the coupling to 1p-1h states by the strong tensor force [8]. A further dramatic result is the ratio of the longitudinal and the transverse spin responses being close to one, while the strong pionic correlations ought to provide a large enhancement in the longitudinal channel [9]. In addition, there appear large short range correlations, which bring the single particle strengths up to highly excited states, seen in the (e,e'p) experiments as caused by the strong tensor force.

All these facts suggest us to look into the possibility of finite pion mean field in medium and heavy nuclear systems. We have been reluctant in doing this, since we have to break the parity and isospin symmetries once the pion mean field becomes finite and have to project out the states with these good quantum numbers in order to get observables. There is also a common sense that pion condensation does not happen in nuclear matter of the saturation density [10, 11]. This fact does not mean, however, that the pion mean field vanishes in finite nuclear system. The pion is a pseudoscalar meson; it couples with a nucleon through the $\vec{\sigma}\cdot\vec{\nabla}$ coupling, and therefore the source term of the pion field needs the parity mixing and the density modulation [12]. In infinite matter, we have to provide this density modulation for the pion to work, which costs energy. In finite nuclear system, we have the nuclear surface and automatically there is the necessary density modulation for the pion to work. Hence, if the pion mean field turns out finite in nuclear system, the pionic correlations utilize the change of the density at the surface. Hence, we would like to name this phenomenon as surface pion condensation.

We start with writing the relativistic meson-nucleon Lagrangian density, which naturally includes the pion term, π;

$$\mathcal{L} = \bar{\psi}[i\gamma^{\mu}\partial_{\mu} - M - g_{\pi}\gamma_5\gamma^{\mu}\tau^a\partial_{\mu}\pi^a - g_{\sigma}\sigma - g_{\omega}\gamma^{\mu}\omega_{\mu} - g_{\rho}\gamma_{\mu}\tau^a\rho^{a\mu} - e\gamma_{\mu}\frac{(1-\tau_3)}{2}A^{\mu}]\psi$$
$$+\frac{1}{2}\partial_{\mu}\pi^a\partial^{\mu}\pi^a - \frac{1}{2}m_{\pi}^2\pi^a\pi^a + \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \frac{1}{2}m_{\sigma}^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4$$
$$-\frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu} + \frac{1}{4}c_3(\omega_{\mu}\omega^{\mu})^2 - \frac{1}{4}G_{\mu\nu}^aG^{a\mu\nu} + \frac{1}{2}m_{\rho}^2\rho_{\mu}^a\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where the field tensors H, G and F for the vector fields are defined through

$$H_{\mu\nu} = \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu}, \tag{2a}$$
$$G_{\mu\nu}^a = \partial_{\mu}\rho_{\nu}^a - \partial_{\nu}\rho_{\mu}^a - g_{\rho}\,\epsilon^{abc}\rho_{\mu}^b\rho_{\nu}^c, \tag{2b}$$
$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{2c}$$

The pion term couples with the nucleon through the pseudo-vector coupling. We take here all the terms used in Ref. [13]. Here, σ denotes the scalar meson, ω the vector meson and ρ the isovector-vector meson. The photon is denoted by A. At the same time, in principle, there should be a coupling of pion with the delta state. In this paper, we do not include this contribution for simplicity of discussion, although this contribution will act constructively for surface pion condensation. We do not consider the tensor coupling term of the ρ meson either in order to concentrate on the pion degree of freedom. All these effects will be worked out in the near future.

We then assume that the expectation value of the pion field is finite. In addition, the nuclear system is isospin-singlet due to the choice of N=Z nuclei and hence the self-consistent Hamiltonian is invariant under the isospin rotation [14]. We choose therefore the finite value of the pion mean field as the z-component; i.e. $a = 0$, without the loss of generality . We write the finite pion mean field as π without the isospin suffix.

For simplicity, we write only the equations of motion for the nucleon, the sigma and pion with only the linear terms. They are written as,

$$[i\gamma^{\mu}\partial_{\mu} - M - g_{\sigma}\sigma - g_{\pi}\vec{\nabla}\pi\gamma_5\vec{\gamma}\tau_0]\psi = 0, \tag{3}$$

for nucleon and

$$(\vec{\nabla}^2 - m_{\pi}^2)\pi = -g_{\pi}\vec{\nabla}\langle\bar{\psi}\gamma_5\vec{\gamma}\tau_0\psi\rangle, \tag{4}$$

and

$$(\vec{\nabla}^2 - m_{\sigma}^2)\sigma = g_{\sigma}\langle\bar{\psi}\psi\rangle, \tag{5}$$

for pion and sigma mesons. Here, the bracket $\langle\cdots\rangle$ above denotes the ground state expectation value. The other mesons follow the same equations of motion as above.

The equations (3) and (4) tell the structure of surface pion condensation. The pion field is generally finite when the source term breaks the parity. The pion field is enhanced by a spacial change of the source term. When the pion field is finite in (3), the nucleon single particle state breaks the parity, which provides again the pion source term finite. The self-consistency provides the converged solution to the above equations.

These equations tell the reason why we have not included the pion mean field until now. The violation of the parity is caused by the pion term in the above Dirac equation for nucleons. Hence, the single particle state can be expressed as

$$\psi_{njm}(x) = \sum_{\kappa} W_{\kappa}^n \phi_{\kappa jm}(x). \tag{6}$$

Here, $\phi_{\kappa jm}$ denote nucleon single particle wave functions with the total angular momentum j and its projection m. We assume the spherical symmetry for the intrinsic state. The summation over κ means the parity mixing, where κ is $\kappa = -(l_{\uparrow}+1)$ for $l_{\uparrow} = j - 1/2$ and $\kappa = l_{\downarrow}$ for $l_{\downarrow} = j + 1/2$. The calculational detail will be provided in the forthcoming publication [15].

We show the numerical results. We take the TM1 parameter set of Ref. [13]. As for the pion-nucleon coupling we take the value of Bonn A potential [16], which corresponds to taking $g_{\pi} = f_{\pi}/m_{\pi}$ and $f_{\pi} \sim 1$. We stress again here that we use all the terms given in the Lagrangian (1). This means that the saturation property is guaranteed and the bulk part of the nucleus tends to have the saturation density. Since we are especially interested in the occurrence of finite pion mean field and want to see its effect under the simplest condition, we neglect the Coulomb term. We calculate the N=Z closed shell nuclei as ^{12}C, ^{16}O, ^{40}Ca, ^{56}Ni, ^{80}Zr, ^{100}Sn and ^{164}Pb.

We show the results in Table I with and without pion mean field. We show the total binding energy (BE), the kinetic energy (KE), the sigma energy (V_{σ}), omega energy (V_{ω}) and pion energy (V_{π}). The finite pion mean field case provides larger total binding energy than the one without pion mean field. The pion energy is attractive, while all the other terms as the kinetic energy, the combined sigma and omega energy do not favor finite pion mean field.

We show the mass number dependence of the pion energy per nucleon in Fig. 1. We mention here that the kinetic energy and the sigma and omega energies are almost constant of the mass number and hence they are volume like as easily recognized from the numbers in Table I. We see, on the other hand, a peculiar behavior in the pion

TABLE I: The contributions of kinetic energy (KE), sigma (V_σ), omega (V_ω) and pion (V_π) energies to the total binding energy (BE) given in MeV. Many N=Z closed shell nuclei are calculated with and without finite pion mean field. ^{12}C, ^{56}Ni, ^{100}Sn and ^{164}Pb are the jj-closed shell nuclei and ^{16}O, ^{40}Ca and ^{80}Zr are the LS-closed shell nuclei.

	With π mean field					Without π mean field			
	BE	KE	V_σ	V_ω	V_π	BE	KE	V_σ	V_ω
^{12}C	116	228	-1387	1196	-108	98	197	-1376	1128
^{16}O	148	227	-1794	1490	-14	148	223	-1811	1498
^{40}Ca	431	576	-4914	4127	-81	427	556	-5003	4164
^{56}Ni	650	955	-7431	6316	-281	626	877	-7622	6340
^{80}Zr	931	1118	-10458	8751	-43	931	1106	-10519	8784
^{100}Sn	1231	1599	-13742	11629	-326	1214	1509	-14006	11689
^{164}Pb	2109	2520	-22913	19352	-409	2094	2409	-23352	19533

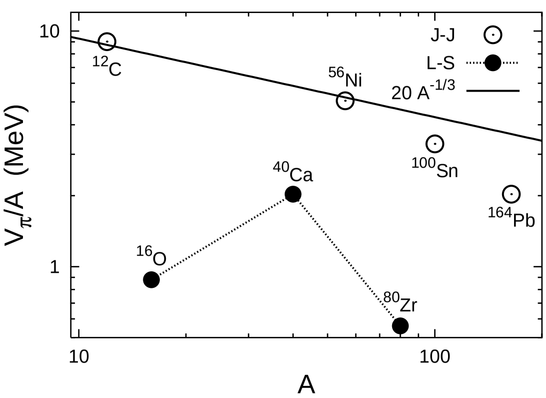

FIG. 1: The pion energy per nucleon as a function of the mass number in the log-log plot. There are two groups; one is for the jj-closed shell nuclei denoted by open circle and the other is for the LS-closed shell nuclei denoted by closed circle. The pion energy per nucleon for the jj-closed shell nuclei decreases monotonically and follows more steeply than $A^{-1/3}$, which is shown by solid line.

energy. The magnitudes of the pion energy are clearly separated into two groups. One group is large and the common feature is jj-closed shell nuclei; the magic number nuclei due to a larger spin-orbit partner (j-upper) being filled. The other group is small and they are LS-closed shell nuclei. The pion energy per nucleon for jj-closed shell nuclei decreases monotonically with the mass number. The rate of the decrease follow more strongly than $A^{-1/3}$. This means that the pion mean field energy behaves in proportion with the nuclear surface or even stronger than that. Hence, we use the word of surface pion condensation. Concerning the separation into two groups for the pion energy; LS-closed and jj-closed shell cases, we shall mention the possible reason later.

We discuss here the role of the pion by performing the parity projection from the symmetry broken intrinsic state. We write the single particle state with mixed parity (6) in a simpler form as

$$|\bar{jm}\rangle = \alpha_j|jm\rangle + \beta_j|\tilde{jm}\rangle. \tag{7}$$

Here, $|\bar{jm}\rangle$ denotes a parity mixed single particle state expressed as a linear combination of $|jm\rangle$, some parity state (we call it as a normal parity state) and $|\tilde{jm}\rangle$, the opposite parity state (abnormal parity state). We write the intrinsic state with these single particle states up to the Fermi surface and with all the magnetic sub-shells filled as

$$\begin{aligned}\Psi =& \prod_{jm}(\alpha_j|jm\rangle + \beta_j|\tilde{jm}\rangle)\\ =& \prod_{jm}\alpha_j|jm\rangle + \sum_{j_1m_1}\prod_{jm\neq j_1m_1}\alpha_j\beta_{j_1}|jm\rangle|\tilde{j_1m_1}\rangle \\ &+ \sum_{j_1m_1j_2m_2}\prod_{jm\neq j_1m_1j_2m_2}\alpha_j\beta_{j_1}\beta_{j_2}|jm\rangle|\tilde{j_1m_1}\rangle|\tilde{j_2m_2}\rangle + \cdots\end{aligned} \tag{8}$$

This intrinsic state has the total spin 0 because all the magnetic sub-shells are filled, but the parity is mixed. The first term, $\prod_{jm}\alpha_j|jm\rangle$, in (8) has the positive parity and corresponds to the ground state in the zeroth order. The second term has the negative parity, since each normal parity state, $|j_1m_1\rangle$, is replaced by an abnormal parity state, $|\tilde{j_1m_1}\rangle$ for all occupied $|j_1m_1\rangle$. Hence, if we say the first term as the 0p-0h state, then the second term is a coherent 1p-1h state with 0^- spin parity. The third term consists of 2p-2h states with a pair of 1p-1h states with 0^- spin parity and therefore has 0^+ spin parity. The next term has three 1p-1h states with 0^- spin parity and therefore has 0^- spin parity and so on.

Hence the positive parity projection P_+ would provides the state with even number of 1p-1h states with 0^- spin parity. $P_+\Psi = |0\rangle + |2p-2h\rangle + |4p-4h\rangle + \cdots$. This means that the positive parity projection provides 2p-2h states as the major correction terms. Hence, surface pion condensation together with parity projection provides the 2p-2h admixture due to the pion exchange interaction as the case of the α particle [6]. The negative parity projection P_- would provide the state with odd number of 1p-1h states with 0^- spin parity. $P_-\Psi = |1p-1h\rangle + |3p-3h\rangle + \cdots$. This is the brother state having the quantum number of 0^- to the 0^+ ground state. The ground state consists of highly correlated particle–hole states and the 0^- state is, therefore, a coherent 1p-1h state compiled on the highly correlated 0^+ ground state.

This demonstration gives us the hint of providing the two groups of the strong pion energy case, the jj-closed shell, and the weak pion energy case, the LS-closed shell.

In the LS-closed shell case, the higher spin-orbit partner in the upper shell above the Fermi surface is not used for the 0^- particle-hole excitations. On the other hand, in the jj-closed shell case, this higher spin-orbit partner state is now filled and is used for the 0^- particle-hole excitations as a hole state. Hence, more attraction is expected for jj-closed shell nuclei than those of the LS-closed shell nuclei. The pion energy is gained largely for the jj-closed shell nuclei, because the number of states to be mixed by the pionic correlations increases and these states contribute coherently to the pion energy. We are, however, not yet clear about the peculiar mass dependence of the LS-closed shell nuclei. This distinct feature of the difference between the LS-closed and the jj-closed shell nuclei can be the consequence of using only the pion correlations.

We discuss here the qualitative consequence of surface pion condensation. First, we discuss the Gamow-Teller (GT) transitions. Without pion condensation, there exist no transitions for LS-closed shell nuclei as ^{16}O, ^{40}Ca and only two transitions, for example, for ^{90}Zr. However, the mixing of parity in the intrinsic state allows transitions of 2p–2h states. This makes the spectrum of the GT transitions with some GT strengths above the two dominant peaks in ^{90}Zr [8]. Hence, naturally we have large strengths in the continuum in the simple mean field theory as the experiment demands.

The longitudinal spin response functions should be largely modified due to pion condensation. The longitudinal spin response is caused by the pionic correlations. Since the large pionic strength is used up to construct the nuclear ground state, the pionic fluctuation oughts to be reduced largely. This should make the spin response in the pion channel weak. This remains to be demonstrated in the future work.

The surface pion condensation provides us with the possibility to describe the short range correlation effect. As seen above in the discussion of the parity projection, the surface pion condensation provides a large amount of the 2p–2h excitations in the nuclear ground state automatically. There should be many other consequences of surface pion condensation in nuclear phenomena. The pairing correlations and the spin-orbit couplings are all surface phenomena and surface pion condensation would couple with these correlations and would provide rich phenomena.

We have discussed the possible occurrence of surface pion condensation in order to understand the recent (p,n) experimental data taken at RCNP. This suggestion is motivated by the missing pion contribution in the discussion of ground states of finite nuclei, while pions are essential for hadron physics. We have made calculations with the inclusion of finite pion mean field and demonstrated the finite results for N=Z closed shell nuclei. We have demonstrated that the pion energy behaves proportional to or even stronger than the nuclear surface. Hence the name surface pion condensation is used in this paper. We have made qualitative discussions on the consequence of surface pion condensation on the Gamow-Teller strengths, the spin response functions and the short range correlations. The large difference of the pion energies in jj-closed shell and LS-closed shell nuclei has been found and may be connected with the mechanism of a part of spin-orbit splitting as due to the tensor force discussed long time ago by Takagi *et al.* and Terasawa [17]. To go further, we have to fix the parameters of the Lagrangian (1), which includes the effect of finite pion mean field and other terms as the delta state and the ρ meson tensor term.

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